

Logics for Machine learning

Gregoire Fournier ¹

¹Department of Mathematics, Statistics, and Computer Science,
University of Illinois Chicago, USA

March 3, 2025



NORTHWESTERN
UNIVERSITY

We want to learn graph invariant function. Graph neural networks (GNNs) are neural network architectures that guarantee invariance by their design.

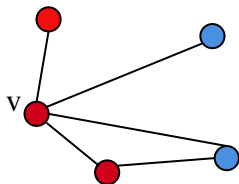
The combination function *comb* is learned, and represented by a neural network. A layer of a GNN acts on a graph, where each vertex v is assigned a state, $\zeta(v)$. It produces an output $\eta(v)$ on each vertex v , given by the equation:

$$\eta(v) := \text{comb}(\zeta(v), \sum_{w \in N_G(v)} \zeta(w))$$

Graph neural networks

Graph neural networks (GNNs) are deep learning architectures for machine learning problems on graphs.

$$\eta(v) := \text{comb}(\zeta(v), \sum_{w \in N_G(v)} \zeta(w))$$



$$\eta(v) := \text{comb}\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix}\right)$$

The combination function *comb* is learned.

The guarded fragment \mathcal{GC} restricts quantifiers to range over the neighbours of the current nodes.

\mathcal{GC}_2 is also known as graded modal logic .

Example 1

The following \mathcal{GC}_2 -formula $\Phi(x)$ says that vertex x has at most 1 neighbour that has more than 10 neighbours with label P_1 :

$$\Phi(x) := \neg \exists^{\geq 2} y (E(x, y) \wedge \exists^{\geq 11} x (E(y, x) \wedge P_1(x)))$$

Theorem 2

Let f be formula in graded modal logic \mathcal{GC}_2 . Then there is a GNN that expresses f .

Theorem 3

Let f be a formula expressible by a GNN and also expressible in first-order logic. Then f is expressible in \mathcal{GC}_2 .

- ▶ Reference: *Elements of Finite Model Theory*, Libkin, 2004
- ▶ Recall compactness: An infinite theory is consistent iff every finite subset is consistent.
- ▶ Recall Downward L-S: If a theory has an infinite model, then it has a countable model as well.

- ▶ Reference: *Elements of Finite Model Theory*, Libkin, 2004
- ▶ Recall compactness: An infinite theory is consistent iff every finite subset is consistent.
- ▶ Recall Downward L-S: If a theory has an infinite model, then it has a countable model as well.

- ▶ Connectivity of arbitrary graphs is not FO-definable, using compactness theorem; consider the sentence

$$\lambda_n := \exists x_1 \dots x_n \neg (E(c_1, x_1) \dots E(x_n, c_2)).$$

- ▶ Compactness fails over finite models: there is a theory T such that
 1. T has no finite models
 2. every finite subset of T has a finite model

Consider the sentence $\lambda_n := \exists x_1 \dots x_n \bigcup_{i,j} x_i \neq x_j$.

Definition 4 (Recursively enumerable set)

A subset L of Σ^* is recursively enumerable if there is a Turing machine that accepts it.

Definition 5 (Recursive set)

A subset L of Σ^* is recursive if there is a Turing machine that decides it.

Theorem 6

The set of logically valid sentences of first-order logic is recursively enumerable. (true in all structures, under all assignments)

But if we consider only finite models, this fails:

Theorem 7 (Trakhtenbrot)

The set of sentences of first-order logic valid in all finite structures is not recursively enumerable.

An important tool for expressibility proofs: EF games

Definition 8 (EF games)

The game is played on two relational structures \mathfrak{A} and \mathfrak{B} , and it has two players, a spoiler and a duplicator. It goes as follows:

- ▶ For n rounds:
 - ▶ The spoiler makes a move by picking an element of \mathfrak{A} or \mathfrak{B} .
 - ▶ The duplicator responds by picking an element in the other structure.
- ▶ The n -rounds game ends in the position $\vec{a} = (a_1, \dots, a_n)$, $\vec{b} = (b_1, \dots, b_n)$. Duplicator wins if: $((\vec{a}, \vec{c}^{\mathfrak{A}}), (\vec{b}, \vec{c}^{\mathfrak{B}}))$ is a partial isomorphism btw. \mathfrak{A} and \mathfrak{B} .

Theorem 9

The following are equivalent:

- ▶ \mathfrak{A} and \mathfrak{B} agree on $FO[k]$.
- ▶ $\mathfrak{A} \equiv_k \mathfrak{B}$ (duplicator has a winning strategy in k -round game).

- ▶ Gaifman: the Gaifman graph, denoted by $G(A)$, has a node for each element of the universe of A , an edge (a_1, a_2) is in $G(A)$ iff $a_1 = a_2$, or there is a relation in A involving a_1 and a_2 .
- ▶ Hanf: A formula f is Hanf-local if there exists a number $d \geq 0$ such that for every structures A, B , for a, b elements of A, B :
The isomorphism type of (A, a) is the same as the isomorphism type of (B, b) up to a distance d on the Gaifman graph
implies that

$$A \models f(a) \text{ iff } B \models f(b)$$

An application of EF games, paper 1

Logics for
Machine learning

Gregoire Fournier

Thesis
introduction

Finite model
Theory
introduction

Finite model
theory paper

Graph NN
explanation
paper

LLM-Law paper

Discussion

Theorem 10

Let $k > 0$, L_1, L_2 be linear orders of length at least 2^k , then $L_1 \equiv_k L_2$ and $L_1 \equiv_{FO[k]} L_2$.

Theorem 11

(F): *Let $q, k > 0$, and let L_1, L_2 be linear orders of length at least $(q + 1)^k$, then $L_1 \equiv_{C^q[k]} L_2$.*

Succinctness on linear orders, paper 1

Logics for
Machine learning

Gregoire Fournier

Thesis
introduction

Finite model
Theory
introduction

Finite model
theory paper

Graph NN
explanation
paper

LLM-Law paper

Discussion

Theorem 12

Let \mathcal{A}_m and \mathcal{A}_n be two linear orders of size $n > m > 0$.

*\mathcal{A}_m and \mathcal{A}_n cannot be distinguished by an
 $FO_3 - (<, succ, min, max)$ sentence of size less than $\frac{\sqrt{m}}{2}$.*

Theorem 13 (**F**)

*\mathcal{A}_m and \mathcal{A}_n cannot be distinguished by an
 $C_3^{(k)} - (<, succ, min, max)$ sentence of size less than $\frac{\sqrt{m}}{k+1}$.*

Open problem: How to characterize the complexity of formulas produced by Graph neural networks?

- ▶ **(F):** EF game characterisation of formula size, on \mathcal{C} with bounded variables, and application to linear orders.
- ▶ **(F):** Application to linear orders of the EF game characterisation of formula size on \mathcal{C} .

Timeline:

- ▶ Started exploring the topic in Summer 2021, independent study in Spring 2022 on this topic.
- ▶ In particular started exploring games in Fall 2022, submitted in late Fall 2023.

Reviews:

- ▶ 2 Accept
- ▶ 2 Accept
- ▶ 0 Borderline

Result: paper rejected, resubmitted to CSL 2025.

Thesis
introduction

Finite model
Theory
introduction

Finite model
theory paper

Graph NN
explanation
paper

LLM-Law paper

Discussion

- ▶ 1 Weak Accept
- ▶ -2 Reject
- ▶ 0 Borderline

Result: paper rejected, resubmitted to LICS 2025.

COMERECGC: common recourse explanation of GNN

See poster.

Logics for
Machine learning

Gregoire Fournier

Thesis
introduction

Finite model
Theory
introduction

Finite model
theory paper

Graph NN
explanation
paper

LLM-Law paper

Discussion

Timeline:

- ▶ Begin working on this topic in Fall 2023 with IDEAL.
- ▶ Submitted after 1 year.
- ▶ Reviewed papers for SDM, ICLR, ICML conferences.

Reviews:

- ▶ 5 Weak Reject
- ▶ 5 Weak Reject
- ▶ 6 Weak Accept
- ▶ 6 Weak Accept

Result: paper rejected, resubmitted to ICML 2025.

The need for interpretability in the legal context

Logics for
Machine learning

Gregoire Fournier

Thesis
introduction

Finite model
Theory
introduction

Finite model
theory paper

Graph NN
explanation
paper

LLM-Law paper

Discussion

- ▶ **Trust and Transparency:** For legal practitioners and other users to be able to understand the model's reasoning process, verify and trust it.
- ▶ **Accountability:** Judicial decisions must be clear and justifiable.
- ▶ **Expanding access to justice:** To make it easier for laypeople to engage with and assess AI-generated legal content.

Pitfalls of LLMs generated legal reasoning

- ▶ **False or misleading legal information:** an LLM may invent laws, precedents, events ...
- ▶ **Lack of interpretability:** What is the LLM's answer based on?
- ▶ **Hard to verify:** Why is the output correct?

Main Inquiry

Can LLMs with *Context Augmentation* and *Chain of Thought* prompting generate **accurate, factual, relevant,** and **comprehensive** legal reasoning for **Landlord-Tenant** problems?

Chain of thought (CoT) [Wei et al., 2023] is a prompting technique that consists in generating intermediate steps to a problem, gradually reaching a final answer. Improvements include:

- ▶ Creating prompts to yield more logical arguments [Press et al., 2023],
- ▶ Adding contextual information at different stages [Trivedi et al., 2022],
- ▶ Leveraging several reasoning paths [Wang et al., 2023].

Context augmentation

The screenshot shows the American Legal Publishing eCodeALP website. The top navigation bar includes the American Legal Publishing logo, a search icon, a login icon, a select language dropdown, and a resources dropdown. The breadcrumb trail indicates the location: IL > Chicago > Municipal Code of ... > 5-12-010 Title, purpose and scope. The left sidebar contains a table of contents for the 5-12-010 Title, with the following items: CHAPTER 5-8 CHICAGO FAIR HOUSING REGULATIONS, CHAPTER 5-10 TENANT OPPORTUNITY TO PURCHASE WOODLAWN PILOT PROGRAM, CHAPTER 5-11 TENANT OPPORTUNITY TO PURCHASE BLOCK (606) DISTRICT PILOT PROGRAM, CHAPTER 5-12 RESIDENTIAL LANDLORD AND TENANTS, and 5-12-010 Title, purpose and scope. The main content area displays the text of the 5-12-010 Title, purpose and scope, which includes sections on Exclusions, Definitions, Tenant responsibilities, Landlord's right of access, Remedies for improper denial of access, and Landlord's responsibility to. The text is formatted with bold headings and subheadings, and includes references to other sections of the code.

Figure: Chicago's Residential Tenant-Landlord Ordinance (RLTO) is 12-page regulation for landlord-tenant relationships.

We ask the LLM to refer solely to text of the RLTO to avoid giving false legal information.

Logics for
Machine learning

Gregoire Fournier

Thesis
introduction

Finite model
Theory
introduction

Finite model
theory paper

Graph NN
explanation
paper

LLM-Law paper

Discussion

Prompt to generate legal reasoning

Input Exposition: *[Provide a detailed description of the legal scenario, including relevant facts, context, and specific issues at stake.]*

Figure: Step 1: Input Exposition

Prompt to generate legal reasoning

Logics for
Machine learning

Gregoire Fournier

Thesis
introduction

Finite model
Theory
introduction

Finite model
theory paper

Graph NN
explanation
paper

LLM-Law paper

Discussion

Input Exposition: *[Provide a detailed description of the legal scenario, including relevant facts, context, and specific issues at stake.]*

Task: Generate structured legal arguments based on the exposition provided.

Figure: Prompt to generate legal reasoning

Tenant and Landlord Laws

Input Exposition: *[Provide a detailed description of the legal scenario, including relevant facts, context, and specific issues at stake.]*

Task: Generate structured legal arguments based on the exposition provided.

Tenant and Landlord Laws considered are the Residential Landlord and Tenant Ordinance of the City of Chicago provided below:

[Text of the RLTO.]

Figure: Context augmentation

Prompt to generate legal reasoning

Input Exposition: *[Provide a detailed description of the legal scenario, including relevant facts, context, and specific issues at stake.]*

Task: Generate structured legal arguments based on the exposition provided.

Tenant and Landlord Laws considered are the Residential Landlord and Tenant Ordinance of the City of Chicago provided below:

[Text of the RLTO.]

Output Format:

Argument 1:

Exposition: Summarize relevant facts from the input that relate to this argument.

Specific Law: Identify a specific law or statute that applies to the scenario.

Why This Law Applies: Explain how and why this law is relevant to the facts presented.

Conclusion: State the conclusion derived from the application of this law to the facts provided.

[Continue generating arguments as necessary, each focusing on a different applicable law.]

Logics for
Machine learning

Gregoire Fournier

Thesis
introduction

Finite model
Theory
introduction

Finite model
theory paper

Graph NN
explanation
paper

LLM-Law paper

Discussion

The output was evaluated by lawyer at the **LCBH**¹, who is an expert in landlord-tenant law, using the metrics:

- ▶ **Accuracy** for a given legal scenario measures how closely the set of generated arguments aligns with the true or expected answer.
- ▶ **Comprehensiveness** measures how well one given argument coherently and concisely addresses the relevant aspects of the input legal scenario regarding the legal requirement cited.
- ▶ **Factuality** assesses whether an argument originates from the text of the RLTO.
- ▶ **Relevance** evaluates whether the argument logically relates to the legal scenario.

Law Center for Better Housing (LCBH)

The LCBH is a *non-profit law firm* that aims at **protecting renters' rights** in the Chicago area.

Rentervention, one of its programs, provides free resources to help tenants enforce their rights, including a *chatbot that helps renters find the correct information* for their issue.

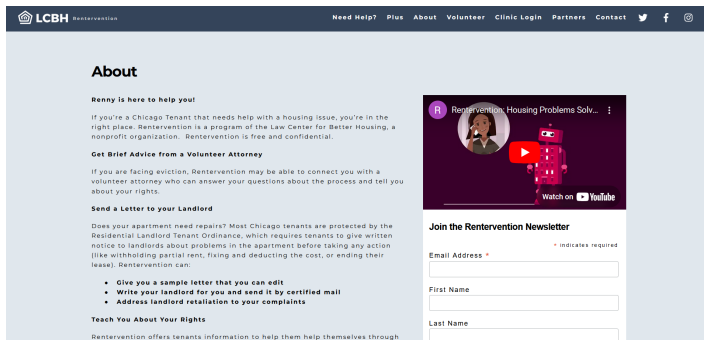
The image is a screenshot of the Rentervention website. At the top, there is a dark navigation bar with the LCBH logo and the word "Rentervention" on the left, and links for "Need Help?", "Plus", "About", "Volunteer", "Clinic Login", "Partners", "Contact", and social media icons for Twitter, Facebook, and Instagram on the right. The main content area has a light blue background. On the left, under the heading "About", there are sections: "Renny is here to help you!" with a paragraph about the program, "Get Brief Advice from a Volunteer Attorney" with a paragraph about connecting with an attorney, "Send a Letter to your Landlord" with a paragraph about the Residential Landlord Tenant Ordinance and a bulleted list of steps (Give you a sample letter, Write your landlord for you, Address landlord retaliation), and "Teach You About Your Rights" with a paragraph about tenant information. On the right, there is a video player titled "Rentervention: Housing Problems Solv..." with a play button and a "Watch on YouTube" link. Below the video is a "Join the Rentervention Newsletter" section with a form for "Email Address" (marked as required), "First Name", and "Last Name".

Figure: Rentervention's website.

Logics for
Machine learning

Gregoire Fournier

Thesis
introduction

Finite model
Theory
introduction

Finite model
theory paper

Graph NN
explanation
paper

LLM-Law paper

Discussion

Table: Scenarios and their sources.

Scenario Number	Description	Source
1	Heating and mold problems	Claude
2	Rent increase	Claude
3	Privacy concerns	Claude
4	Property damage	Claude
5	Cockroach infestation	Rentervention
6	Eviction notice	Rentervention
7	Landlord asking about crime	Rentervention
8	Roof leaking	Rentervention
9	Landlord taking photos inside	Rentervention
10	Wear and tear	Authors

Example 1: Input

2 Scenario 2: Rent increase

2.1 Input

I began renting my one-bedroom apartment on May 1st, 2022, with a reasonable rent for the area. On April 15th, 2023, about two weeks before my lease was set to renew, I received an email from my landlord, Mr. Johnson, informing me that he would be raising the rent by 8% for the upcoming year due to increasing property taxes and maintenance costs.

Caught off guard by this increase, I spent the next few days looking at other rental listings in the area and found that the new rate was still competitive for the neighborhood. On April 20th, I emailed Mr. Johnson to ask if there was any room for negotiation. He responded on April 22nd, offering to reduce the increase to 7% if I signed a two-year lease instead of one.

Example 1: Output

- **Argument 1:**

- **Exposition:** The tenant received an email from the landlord on April 15th, 2023, informing them of an 8% rent increase, citing increased property taxes and maintenance costs. The tenant found the new rate competitive but sought to negotiate. On April 22nd, the landlord offered to reduce the increase to 7% if the tenant signed a two-year lease.
- **Specific Law:** Chicago Residential Landlord and Tenant Ordinance (RLTO) Section 5-12-130(j)(2)
- **Why This Law Applies:** Section 5-12-130(j)(2) stipulates that for any residential tenancy of six months to three years, the landlord must notify the tenant in writing at least 60 days before the termination date of the rental agreement if they intend to increase the rent.
- **Conclusion:** The landlord's email on April 15th, notifying the tenant of a rent increase effective May 1st, does not comply with the 60-day notice requirement. Therefore, the tenant may argue that the rent increase is invalid due to insufficient notice.

This argument is **accurate, relevant, factual** and **comprehensive**. This is what a lawyer would advise their client.

Example 1: Output

- **Argument 2:**

- **Exposition:** The landlord provided a revised offer to reduce the rent increase to 7% if the tenant signed a two-year lease.
- **Specific Law:** Chicago Residential Landlord and Tenant Ordinance (RLTO) Section 5-12-140(a)
- **Why This Law Applies:** Section 5-12-140(a) states that no rental agreement may provide that the tenant agrees to waive or forgo rights, remedies, or obligations provided under this chapter.
- **Conclusion:** The tenant could argue that the landlord's offer to reduce the rent increase contingent on signing a two-year lease may be seen as an attempt to make the tenant waive their right to proper notice of rent increase, thus violating Section 5-12-140(a).

- ▶ This argument is **relevant** and **factual**.
- ▶ However, the interpretation of the law is incorrect; therefore, it is **not accurate nor comprehensive**.

Example 1: Output

- **Argument 3:**

- **Exposition:** The tenant has been renting since May 1st, 2022, and he landlord cited increased property taxes and maintenance costs as reasons for the rent increase.
- **Specific Law:** Chicago Residential Landlord and Tenant Ordinance (RLTO) Section 5-12-150
- **Why This Law Applies:** Section 5-12-150 prohibits landlords from taking retaliatory actions against tenants for exercising their rights or seeking repairs.
- **Conclusion:** If the tenant had previously requested repairs or exercised other rights, the tenant could argue that the rent increase is retaliatory, especially if the landlord cannot substantiate the claimed increases in property taxes and maintenance costs.

- ▶ The argument is **factual**.
- ▶ It is **comprehensive**, since the reasoning is sound.
- ▶ However, it is **neither relevant nor accurate**, as it ignores the main problem to invent another one.

Key Contributions:

- ▶ We employed *Context Augmentation* and *Chain of Thought (CoT) instructions* with GPT-4o to generate legal arguments for **Landlord-Tenant** issues.
- ▶ We demonstrated the potential of LLMs to produce **accurate, relevant, factual, and comprehensive** legal reasoning.

Identified Limitations:

- ▶ Inability to identify legal issues beyond the provided context.
- ▶ Challenges in assessing the **relevance** of generated arguments.

Timeline:

- ▶ Started working on the topic in late-Spring 2024, was submitted at the end of the Summer.

Reviews:

- ▶ -1 Accept
- ▶ -1 Accept
- ▶ -1 Borderline
- ▶ +2 Accept

Result: paper accepted in the poster format.

What to take from this presentation?

What to do during your PhD:

- ▶ Opportunity to learn and look into a lot of different topics.
- ▶ Set up your own projects and collaborations.

What is a PhD thesis?:

- ▶ Developing your take, your point of view on the topics of your choice.
- ▶ You get to set the rules and boundaries.

Conclusion

Key points:

- ▶ Think about what you want to do after your thesis.
- ▶ Do not hesitate to reach out to other faculty at UIC, IDEAL, or in your or your advisor network!
- ▶ Research: finding the right tool to apply to the right problem.


Unique profile:

- ▶ Number of new Maths and Stats PhD per year: $\approx 2k2$ (2023)
- ▶ Number of new CS PhD per year: $\approx 2k7$ (2023)
- ▶ Number of STEM bachelor degrees: $\approx 500k$ (2021)

-  Press, O., Zhang, M., Min, S., Schmidt, L., Smith, N. A., and Lewis, M. (2023).

Measuring and narrowing the compositionality gap in language models.

In Findings of the Association for Computational Linguistics: EMNLP.


-  Trivedi, H., Balasubramanian, N., Khot, T., and Sabharwal, A. (2022).

Interleaving retrieval with chain-of-thought reasoning for knowledge-intensive multi-step questions.

-  Wang, X., Wei, J., Schuurmans, D., Le, Q. V., Chi, E. H., Narang, S., Chowdhery, A., and Zhou, D. (2023).

Self-consistency improves chain of thought reasoning in language models.

In *The Eleventh International Conference on Learning Representations, ICLR*.

-  Wei, J., Wang, X., Schuurmans, D., Bosma, M., Ichter, B., Xia, F., Chi, E., Le, Q., and Zhou, D. (2023).

Chain-of-thought prompting elicits reasoning in large language models.